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ON THE QUALITATIVE ANALYSIS OF CERTAIN FLOTATION PROCESSES

Daniela D. Parashkevova
Department of Automation of Mining Production, Faculty of Mine Electromechanics, University of Mining and Geology "St. I. Rilski", 1700 Sofia, BULGARIA

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ABSTRACT

The present paper is devoted to the qualitative analysis of certain flotation processes describing by a first order hyperbolic system of partial differential equations. The system in question is like telegrapher equations. That is why, we use the methods for examining the transmission lines set out in the papers mentioned in the References. We formulate a mixed problem for this system with boundary conditions corresponding to the processes in the flotation cameras. We present the mixed problem for the hyperbolic system in a suitable operator form and prove an existence of generalized solution by fixed point method. One can obtain an explicit approximated solution as a step in the sequence of successive approximations.

KEYWORDS: Camera flotation, Qualitative analysis, Hyperbolic system, Operator presentation, Successive approximations, Fixed point method.

1. INTRODUCTION

A lot of papers have been devoted to the investigation of flotation processes. We mention just (Cipriano; Parashkevova, 2006; Prozuto, 1987; Speedy et al., 1970; Loveday et al., 1995; Cienski et al., 1981; Finch et al., 1990; Azgomi et al., 2007; **Sbárbaro** et al., 2010; Rubinstein et al., 1980). Here we study a system describing camera flotation process considered in (Rubinstein et al., 1980). From mathematical point of view, it is a system of hyperbolic type just like lossless transmission lines investigated in (Angelov, 2014):

$$\begin{aligned}
&\frac{\partial C_B(x,t)}{\partial t} = k_1 C_P(x,t) - k_2 C_B(x,t) - V_B \frac{\partial C_B(x,t)}{\partial x}\\
&\frac{\partial C_P(x,t)}{\partial t} = -k_1 C_P(x,t) + k_2 C_B(x,t) + V_P \frac{\partial C_P(x,t)}{\partial x}\\
&(x,t) \in \Pi = \{(x,t) \in R^2 : (x,t) \in [0,H] \times [0,T_0]\}
\end{aligned} \quad (1)$$

Here $C_P(x,t)$ is the mineral concentration in the liquid, $C_B(x,t)$ is the mineral concentration on the bubbles, k_1 and k_2 are prescribed kinetic constants describing particle transitions from one phase to another, $H>0$ is the height of the camera and $[0,T_0]$ is prescribed time interval; the constant $V_P > 0$ is a particle sedimentation rate, the constant $V_B > 0$ - the bubble lifting speed. It is known that $V_B >> V_P$ (cf. Rubinstein et al., 1980). The process in the camera are such that V_P is directed from top to bottom, while V_B the speed of the bubbles is directed upwards.

For system (1) one can formulate the following mixed (initial-boundary value) problem: to find the unknown concentration functions $C_P(x,t)$ and $C_B(x,t)$ in Π satisfying initial conditions

$$C_B(x,0) = 0\,;\; C_P(x,0) = C_{P0}, \quad (2)$$

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where $C_{P0} = const. > 0$ is a prescribed initial concentration and boundary conditions

$$C_B(0,t) = 0, \;\; C_P(0,t) = C_{P0} = const. > 0 \qquad t \in [0, T_0]. \tag{3}$$

In (Angelov, 2014) the mathematical methods for investigation of transmission lines and some applications (cf. also Angelov et al., 2017) are developed. The methods from (Angelov, 2014) in (Angelov, 2016; Angelov, 2016; Angelov, 2019; Angelov, 2015; Angelov, 2015; Angelov, 2019) is generalized to study various problems for transmission line systems without Heaviside condition. It turns out that this method to our flotation problem is applicable. We present the mixed problem for the above hyperbolic system in an operator form. Choosing a suitable function space, we prove existence theorems for (1) - (3) by fixed point method (cf. Angelov, 2009). Finally, we demonstrate a simple way to obtain successive approximations tending to the solution of our problem. Introduce denotations

$$U = \begin{bmatrix} C_B(x,t) \\ C_P(x,t) \end{bmatrix}, \quad \frac{\partial U}{\partial t} = \begin{bmatrix} \dfrac{\partial C_B(x,t)}{\partial t} \\ \dfrac{\partial C_P(x,t)}{\partial t} \end{bmatrix}, \quad \frac{\partial U}{\partial x} = \begin{bmatrix} \dfrac{\partial C_B(x,t)}{\partial x} \\ \dfrac{\partial C_P(x,t)}{\partial x} \end{bmatrix}, A = \begin{bmatrix} V_B & 0 \\ 0 & -V_P \end{bmatrix},$$

Then (1) can be rewrite as

$$\frac{\partial C_B(x,t)}{\partial t} + V_B \frac{\partial C_B(x,t)}{\partial x} = k_1 C_P(x,t) - k_2 C_B(x,t)$$

$$\frac{\partial C_P(x,t)}{\partial t} - V_P \frac{\partial C_P(x,t)}{\partial x} = -k_1 C_P(x,t) + k_2 C_B(x,t)$$

or in a matrix form

$$\begin{bmatrix} \dfrac{\partial C_B(x,t)}{\partial t} \\ \dfrac{\partial C_P(x,t)}{\partial t} \end{bmatrix} + \begin{bmatrix} V_B & 0 \\ 0 & -V_P \end{bmatrix} \begin{bmatrix} \dfrac{\partial C_B(x,t)}{\partial x} \\ \dfrac{\partial C_P(x,t)}{\partial x} \end{bmatrix} = \begin{bmatrix} k_1 & -k_2 \\ -k_1 & k_2 \end{bmatrix} \begin{bmatrix} C_B(x,t) \\ C_P(x,t) \end{bmatrix}$$

or

$$\frac{\partial U}{\partial t} + A \frac{\partial U}{\partial x} = \Gamma U. \tag{4}$$

Here the matrix $A = \begin{bmatrix} V_B & 0 \\ 0 & -V_P \end{bmatrix}$ is in a diagonal form. Therefore the characteristic roots are $\lambda_1 = V_B, \lambda_2 = -V_P$. So we are able to formulate the main problem of the paper:
To solve the hyperbolic system (4) satisfying initial conditions

$$C_B(x,0)=0,\quad C_P(x,0)=C_{P0}=const.>0\quad x\in[0,H] \tag{5}$$

and boundary conditions

$$C_B(0,t)=0,\quad C_P(0,t)=C_{P0},\quad t\in[0,T_0];\quad C_B(H,t)=C_{BH},\quad C_P(H,t)=C_{PH},\quad t\in[0,T_0]$$

$C_B(H,t)+C_P(H,t)=\kappa(t)<1$, where $\overline{\kappa}=\sup\{\kappa(t):t\in[0,T_0]\}<1$.

2. AN OPERATOR FORMULATION OF THE MIXED PROBLEM

The mixed problem is: to find a solution $(C_P(x,t)\ ,\ C_B(x,t))$ of the following system.
Following (Angelov, 2016; Angelov, 2016; Angelov, 2019; Angelov, 2015; Angelov, 2015; Angelov, 2019) we consider the Cauchy problem for the characteristics:

$$d\xi/d\tau=V_B,\ \xi(t)=x\quad \text{for each } (x,t)\in\Pi\ \Rightarrow\ \varphi_B(\tau;x,t)=V_B\tau+x-V_Bt\,, \tag{6}$$

$$d\eta/d\tau=-V_P,\ \eta(t)=x\quad \text{for each } (x,t)\in\Pi\ \Rightarrow\ \varphi_P(\tau;x,t)=-V_P\tau+x+V_Pt\,. \tag{7}$$

Functions $\lambda_B(x,t)=V_B>0$ and $\lambda_P(x,t)=-V_P<0$ are continuous ones and imply a uniqueness to the left from t of the solution $\xi=\varphi_B(t;x,t)$ of $d\xi/dt=V_B$, $\xi(t)=x$ and respectively $\eta=\varphi_P(t;x,t)$ of $d\eta/dt=-V_P$, $\eta(t)=x$.

Denote by $\chi_B(x,t)$ the smallest value of τ such that the solution $\varphi_B(\tau;x,t)=V_B\tau+x-V_Bt$ of (6) still belongs to Π and by $\chi_P(x,t)$ − the respective value of τ for the solution $\varphi_P(\tau;x,t)=-V_P\tau+x+V_Pt$ of (7).
If $\chi_B(x,t)>0$ then $\varphi_B(\chi_B(x,t);x,t)=0$ or $\varphi_B(\chi_B(x,t);x,t)=H$ and respectively if $\chi_P(x,t)>0$ then $\varphi_P(\chi_P(x,t);x,t)=0$ or $\varphi_P(\chi_P(x,t);x,t)=H$. In our case

$$\chi_B(x,t)=\begin{cases} t-\dfrac{x}{V_B} & \text{for } V_Bt-x>0\\ 0 & \text{for } V_Bt-x\le 0\end{cases}\qquad \chi_P(x,t)=\begin{cases} t-\dfrac{H-x}{V_P} & \text{for } V_Pt+x-H>0\\ 0 & \text{for } V_Pt+x-H\le 0\end{cases}.$$

Remark: We notice that $0\le\chi_B(x,t)\le t,\ 0\le\chi_P(x,t)\le t$.
It is easy to see that

$$\varphi_B(\tau;x,t)=V_B\tau+x-V_Bt\ \Rightarrow\ \varphi_B(0;x,t)=x-V_Bt\,;$$

$$\varphi_P(\tau;x,t)=-V_P\tau+x+V_Pt\ \Rightarrow\ \varphi_P(0;x,t)=x+V_Pt\,.$$

Introduce the sets

$$\Pi_{in,B}=\{(x,t)\in\Pi:\ \chi_B(x,t)=0\}\equiv\{(x,t)\in\Pi:x-V_Bt\ge 0\},$$

$$\Pi_{in,P}=\{(x,t)\in\Pi:\ \chi_P(x,t)=0\}\equiv\{(x,t)\in\Pi:V_Pt+x-H\ge 0\},$$

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$$\Pi_{0B} = \{(x,t) \in \Pi : \chi_B(x,t) > 0,\ \ \varphi_B(\chi_B(x,t);x,t) = V_B(V_B t - x)/V_B + x - V_B t = 0\},$$

$$\Pi_{HB} = \{(x,t) \in \Pi : \chi_B(x,t) > 0,\ \ \varphi_B(\chi_B(x,t);x,t) = V_B(V_B t - x)/V_B + x - V_B t = H\} = \varnothing,$$

$$\Pi_{0P} = \left\{(x,t) \in \Pi : \chi_P(x,t) > 0,\ \ \varphi_P(\chi_P(x,t);x,t) = -V_P\left(t - \frac{H-x}{V_P}\right) + x + V_P t = 0\right\} = \varnothing,$$

$$\Pi_{HP} = \left\{(x,t) \in \Pi:\ \chi_P(x,t) > 0,\ \ \varphi_P(\chi_P(x,t);x,t) = -V_P\left(t - \frac{H-x}{V_P}\right) + x + V_P t = H\right\}.$$

Prior to present the mixed problem in operator form we introduce

$$\Phi_B(C_B,C_P)(x,t) = \begin{cases} C_{B0}(x - V_B t),\ (x,t) \in \Pi_{in,B} \\ \Phi_{0B}(C_B,C_P)(\chi_B(x,t)), (x,t) \in \Pi_{0B} \end{cases} = \begin{cases} 0,\ (x,t) \in \Pi_{in,B} \\ C_{B0}\ \ ,(x,t) \in \Pi_{0B} \end{cases}$$

and

$$\Phi_P(C_B,C_P)(x,t) = \begin{cases} C_{P0}(x + V_P t)\ ,\ (x,t) \in \Pi_{in,P} \\ \kappa(t) - C_B(H,\chi_P(x,t)),\ (x,t) \in \Pi_{0P} \end{cases}.$$

So we assign to the above mixed problem the following system of operator equations

$$C_B(x,t) = 0, \quad (x,t) \in \Pi_{in,B};$$

$$C_B(x,t) = C_{B0} + \int_{t-\frac{x}{V_B}}^{t} k_1 C_B(\chi_B(x,t),s) - k_2 C_P(\chi_B(x,t),s)ds, \quad (x,t) \in \Pi_{0B}$$

$$C_P(x,t) = C_{P0}, \quad (x,t) \in \Pi_{in,P} \tag{8}$$

$$C_P(H,t) = \kappa(t) - C_B(H,\chi_P(x,t)) + \int_{t-\frac{H-x}{V_P}}^{t} \left(-k_1 C_B(\chi_P(x,t),s) + k_2 C_P(\chi_P(x,t),s)\right)ds, \qquad (x,t) \in \Pi_{0P}.$$

Introduce the function sets:

$$M_B = \left\{C_B \in C([0,H]\times[0,T_0]) : |C_B(x,t)| \le \hat{C}_B e^{\mu t}, x \in [0,H]\right\},$$

$$M_P = \left\{C_P \in C([0,H]\times[0,T_0]) : |C_P(x,t)| \le \hat{C}_P e^{\mu t}, x \in [0,H]\right\},$$

where $\hat{C}_B, \hat{C}_P$ and μ are positive constants.

It is easy to verify that the set $M_B \times M_P$ turns out into a complete metric space with respect to the metric:

$$\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P)) = \max\left\{\rho(C_B,\overline{C}_B),\ \rho(C_P,\overline{C}_P)\right\}, \text{ where}$$
$$\rho(C_B,\overline{C}_B) = \sup\left\{e^{-\mu t}\left|C_B(x,t)-\overline{C}_B(x,t)\right| : (x,t)\in[0,H]\times[0,T_0]\right\},$$
$$\rho(C_P,\overline{C}_P) = \sup\left\{e^{-\mu t}\left|C_P(x,t)-\overline{C}_P(x,t)\right| : (x,t)\in[0,H]\times[0,T_0]\right\}.$$

Now we define an operator $T=(T_B,T_P): M_B \times M_P \to M_B \times M_P$ by the formulas

$$T_B(C_B,C_P)(x,t) := 0,\ (x,t)\in \Pi_{in,B},$$
$$T_B(C_B,C_P)(x,t) := \quad C_{B0} + \int_{t-\frac{x}{V_B}}^{t} k_1 C_B(\chi_B(x,s),s) - k_2 C_P(\chi_B(x,s),s)ds,\ (x,t)\in\Pi_{0B};$$
$$T_P(C_B,C_P)(x,t) := C_{0P},\quad (x,t)\in \Pi_{in,P},$$
$$T_P(C_B,C_P)(x,t) := \kappa(t) - C_B(H,\chi_P(x,t)) + \int_{t-\frac{H-x}{V_P}}^{t}\left(-k_1 C_B(\chi_P(x,s),s) + k_2 C_P(\chi_P(x,s),s)\right)ds,$$
$$(x,t)\in\Pi_{0P}.$$

3. EXISTENCE THEOREM

We call a generalized solution (C_B,C_P) of (4), (5) if (C_B,C_P) is a solution of integral equations (8). The main purpose of the section is to prove an existence of solution of (8).

Theorem 1. Let the following conditions be fulfilled for sufficiently large $\mu>0$:

1) $\dfrac{(1+k_1)\hat{C}_B + k_2\hat{C}_P}{\mu} \le \hat{C}_B$; **2)** $\hat{\kappa} + \hat{C}_B e^{-\mu\frac{\varepsilon}{V_P}} + \dfrac{k_1\hat{C}_B + k_2\hat{C}_P}{\mu} \le \hat{C}_P$;

3) $e^{-\mu\frac{\varepsilon}{V_P}} + \dfrac{k_1+k_2}{\mu} < 1$; **4)** $C_{B0}\le\hat{C}_B$; $C_{0P}\le\hat{C}_P$; **5)** $\overline{\kappa} = \sup\left\{\kappa(t) : t\in[0,T_0]\right\} < 1$.

Then there exists a unique generalized solution of (8) on the set $[0,H-\varepsilon]\times[0,T_0]$, where $0<\varepsilon<H$.

Proof: We show that the operator $T=(T_B,T_P): M_B\times M_P \to M_B\times M_P$ above introduced maps the set $M_B\times M_P$ into itself. We notice that $T_B(C_B,C_P)(x,t)$ and $T_P(C_B,C_P)(x,t)$ are continuous functions.
First we have to show that $\left|T_B(C_B,C_P)(x,t)\right| \le \hat{C}_B e^{\mu t}$, $\left|T_P(C_B,C_P)(x,t)\right| \le \hat{C}_P e^{\mu t}$.

Indeed, $\left|\Phi_B(C_B,C_P)(x,t)\right| = 0$ and therefore

$$\left|T_B(C_B,C_P)(x,t)\right| \le C_{B0} + \int_{t-\frac{x}{V_B}}^{t} \left(k_1\left|C_B(\chi_B,s)\right| + k_2\left|C_P(\chi_B,s)\right|\right)ds \le$$

$$\le \left((1+k_1)\hat{C}_B + k_2\hat{C}_P\right)\int_{t-\frac{x}{V_B}}^{t} e^{\mu s}ds \le \left((1+k_1)\hat{C}_B + k_2\hat{C}_P\right)\frac{e^{\mu t} - e^{\mu(t-V_B x)}}{\mu} \le \frac{(1+k_1)\hat{C}_B + k_2\hat{C}_P}{\mu}e^{\mu t} \le \hat{C}_B e^{\mu t};$$

$$\left|T_P(C_B,C_P)(x,t)\right| \le \kappa(t) + C_B(H,\chi_P(x,t)) + \int_{t-\frac{H-x}{V_P}}^{t} k_1\left|C_B(\chi_P,s)\right| + k_2\left|C_P(\chi_P,s)\right|ds \le$$

$$\le \hat{\kappa}e^{\mu t} + \hat{C}_B e^{\mu\left(t-\frac{H-x}{V_P}\right)} + \frac{k_1\hat{C}_B + k_2\hat{C}_P}{\mu}e^{\mu t} \le \left(\hat{\kappa} + \hat{C}_B e^{-\mu\frac{H-x}{V_P}} + \frac{k_1\hat{C}_B + k_2\hat{C}_P}{\mu}\right)e^{\mu t} \le$$

$$\le \left(\hat{\kappa} + \hat{C}_B e^{-\mu\frac{\varepsilon}{V_P}} + \frac{k_1\hat{C}_B + k_2\hat{C}_P}{\mu}\right)e^{\mu t} \le \hat{C}_P e^{\mu t}$$

for sufficiently large $\mu > 0$.

The operator T is contractive one. Indeed,

$$\left|T_B(C_B,C_P)(x,t) - T_B(\overline{C}_B,\overline{C}_P)(x,t)\right| \le$$

$$\le \int_{t-\frac{x}{V_B}}^{t} k_1\left|C_B(V_B s + x - V_B s, s) - \overline{C}_B(V_B s + x - V_B s, s)\right| + k_2\left|C_P(V_B s + x - V_B s, s) - \overline{C}_P(V_B s + x - V_B s, s)\right|ds \le$$

$$\le \rho\left(C_B,\overline{C}_B\right)k_1\int_{t-\frac{x}{V_B}}^{t} e^{\mu s}ds + \rho\left(C_P,\overline{C}_P\right)k_2\int_{t-\frac{x}{V_B}}^{t} e^{\mu s}ds \le \max\left\{\rho\left(C_B,\overline{C}_B\right),\rho\left(C_P,\overline{C}_P\right)\right\}(k_1+k_2)\frac{e^{\mu t} - e^{\mu(t-V_B x)}}{\mu} \le$$

$$\le e^{\mu t}\rho\left((C_B,C_P),(\overline{C}_B,\overline{C}_P)\right)\frac{k_1+k_2}{\mu}.$$

Consequently

$$\rho\left(T_B(C_B,C_P),T_B(\overline{C}_B,\overline{C}_P)\right) \le \frac{k_1+k_2}{\mu}\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P)).$$

For the second component we have

$$\left|T_P(C_B,C_P)(x,t) - T_P(\overline{C}_B,\overline{C}_P)(x,t)\right| \le \left|C_B(H,\chi_P(x,t)) - \overline{C}_B(H,\chi_P(x,t))\right| +$$

$$+ \int_{t-\frac{H-x}{V_P}}^{t} k_1\left|C_B(\chi_P,s) - \overline{C}_B(\chi_P,s)\right| + k_2\left|C_P(\chi_P,s) - \overline{C}_P(\chi_P,s)\right|ds \le$$

$$\le \rho\left(C_B,\overline{C}_B\right)e^{\mu\chi P}+\rho\left(C_B,\overline{C}_B\right)k_1\int_{t-\frac{H-x}{V_P}}^{t}e^{\mu s}ds+\rho\left(C_P,\overline{C}_P\right)k_2\int_{t-\frac{H-x}{V_P}}^{t}e^{\mu s}ds\le$$

$$\le \rho\left(C_B,\overline{C}_B\right)e^{\mu\left(t-\frac{H-x}{V_P}\right)}+\rho\left(C_B,\overline{C}_B\right)k_1\int_{t-\frac{H-x}{V_P}}^{t}e^{\mu s}ds+\rho\left(C_P,\overline{C}_P\right)k_2\int_{t-\frac{H-x}{V_P}}^{t}e^{\mu s}ds\le$$

$$\le e^{\mu t}\rho\left(C_B,\overline{C}_B\right)e^{-\mu\frac{H-x}{V_P}}+\rho\left(C_B,\overline{C}_B\right)k_1\frac{e^{\mu t}-e^{\mu\left(t-\frac{H-x}{V_P}\right)}}{\mu}+\rho\left(C_P,\overline{C}_P\right)k_2\frac{e^{\mu t}-e^{\mu\left(t-\frac{H-x}{V_P}\right)}}{\mu}\le$$

$$\le e^{\mu t}\left[\rho\left(C_B,\overline{C}_B\right)e^{-\mu\frac{H-x}{V_P}}+\rho\left(C_B,\overline{C}_B\right)k_1\frac{1-e^{-\mu\frac{H-x}{V_P}}}{\mu}+\rho\left(C_P,\overline{C}_P\right)k_2\frac{1-e^{-\mu\frac{H-x}{V_P}}}{\mu}\right]\le$$

$$\le e^{\mu t}\left(e^{-\mu\frac{H-x}{V_P}}+\frac{k_1}{\mu}+\frac{k_2}{\mu}\right)\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P))\le\left(e^{-\mu\frac{\varepsilon}{V_P}}+\frac{k_1+k_2}{\mu}\right)\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P))$$

which implies

$$\rho\left(T_P(C_B,C_P),T_P(\overline{C}_B,\overline{C}_P)\right)\le\left(e^{-\mu\frac{\varepsilon}{V_P}}+\frac{k_1+k_2}{\mu}\right)\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P)).$$

Since

$$\rho\left(T_B(C_B,C_P),T_B(\overline{C}_B,\overline{C}_P)\right)\le\rho\left((C_B,C_P),(\overline{C}_B,\overline{C}_P)\right)\frac{k_1+k_2}{\mu}$$

and

$$\rho\left(T_P(C_B,C_P),T_P(\overline{C}_B,\overline{C}_P)\right)\le\rho\left((C_B,C_P),(\overline{C}_B,\overline{C}_P)\right)\left(e^{-\mu\frac{\varepsilon}{V_P}}+\frac{k_1+k_2}{\mu}\right),$$

it follows

$$\max\left\{\rho\left(T_B(C_B,C_P),T_B(\overline{C}_B,\overline{C}_P)\right),\rho\left(T_P(C_B,C_P),T_P(\overline{C}_B,\overline{C}_P)\right)\right\}\le\rho\left((C_B,C_P),(\overline{C}_B,\overline{C}_P)\right)\left(e^{-\mu\frac{\varepsilon}{V_P}}+\frac{k_1+k_2}{\mu}\right)$$

that is,

$$\rho\left(\left(T_B(C_B,C_P),T_P(C_B,C_P)\right),\left(T_B(\overline{C}_B,\overline{C}_P),T_P(\overline{C}_B,\overline{C}_P)\right)\right)\le\left(e^{-\mu\frac{\varepsilon}{V_P}}+\frac{k_1+k_2}{\mu}\right)\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P))$$

and in this way we have shown that T is a contractive operator. The fixed point of T is a solution of the mixed problem above formulated.
The main Theorem is thus proved.

4. CONCLUSION REMARKS

Here we show the process of obtaining of successive approximations. We begin with the first step choosing $C_B^{(0)}(x,t) = C_{B0}$, $C_P^{(0)}(x,t) = C_{P0}$. Then

$$C_B^{(n+1)}(x,t) = C_{B0} + \int_{t-\frac{x}{V_B}}^{t} k_1 C_B^{(n)}(\chi_B(x,t),s) - k_2 C_P^{(n)}(\chi_B(x,t),s)ds, \quad (x,t) \in \Pi_{0B}$$

$$C_P^{(n+1)}(H,t) = \kappa(t) - C_B^{(n)}(H,\chi_P(x,t)) + \int_{t-\frac{H-x}{V_P}}^{t} \left(-k_1 C_B^{(n)}(\chi_P(x,s),s) + k_2 C_P^{(n)}(\chi_P(x,s),s)\right)ds, \quad (x,t) \in \Pi_{0P}.$$

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